1. A fair coin is flipped twice and it is known that at least one head is observed. What is the probability of getting two heads ?

   ____

2. Given n indistinguishable particles and m (> n) distinguishable boxes, we place at random each particle in one of the boxes. Find the probability that in n preselected boxes, one and only one particle will be found ?
   a. $n! / (m^n)$
   b. $(m-1)!\, n! \,/\, (m+n-1)!$
   c. $1/(m^n)$
   d. 1 / m
   e. None

3. Which of the following statements are true? (select all statements that are true)
   a. If A and B are two rank 1 matrices then the rank of their sum A + B can never be greater than 1.
   b. Any rank 1 matrix $A_{m \times n}$ can always be written as uv^T where $u \in R^m$ and $v \in R^n$.
   c. If A and B are two rank-1 matrices then the rank of their product AB can never be greater than 1.
   d. If A is a m × p matrix and B is a p × n matrix then if rank(A) ≤ p (always) and rank(B) ≤ p (always) but the rank of AB can be greater than p.

4. What value should we subtract from the diagonal of the given matrix, such that the determinant of the matrix is zero ? __________

$$A = \begin{bmatrix} 0.1 & 0.15 & 0.35 & 0.4 \\ 0.2 & 0 & 0.45 & 0.35 \\ 0.1 & 0.7 & 0.1 & 0.1 \\ 0.3 & 0.1 & 0.5 & 0.1 \end{bmatrix}$$

5. X is a uniform distribution random variable with support in [-2, 2] U [99.5, 100.5]. The mean of ***X*** is _______

6. Consider the matrix **X** whose eigenvalues are 1, -1 and 3. Then Trace of $\mathbf{X}^3 - 3\mathbf{X}^2$ is _____

7. A new test has been developed to determine whether a given student is overstressed. This test is 95% accurate if the student is not overstressed, but only 75% accurate if the student is overstressed. It is known that 40% of all students are over-stressed. Given that a particular student tests negative for stress, what is the probability that the test result is correct ? ______

8. If A is an m×n matrix, find dim(R(A)) + dim(C(A)) + dim(N(A)) + dim($N(A^T)$),
   where R(.) is defined as the row space of the matrix,
   C(.) is defined as the column space of the matrix,
   N(.) is defined as null space of the matrix,

   a. m + n
   b. 2(m+n)
   c. mn
   d. mn + m + n
   e. None

9. Calculate $\dfrac{dy}{dx}$, if

$$z = x^2 + x^3 + \sqrt{1-x^2}$$
$$4z = e^{-2y} + 2\sin y$$

a. $y\left(2x + 3x^2 - \dfrac{x}{\sqrt{1-x^2}}\right)$

b. $\dfrac{2x + 3x^2 + \dfrac{x}{\sqrt{1-x^2}}}{e^{-2y} + 2\cos y}$

c. $\dfrac{4x + 6x^2 - \dfrac{2x}{\sqrt{1-x^2}}}{-e^{-2y} + \cos y}$

d. $\dfrac{2x + 3x^2 - \dfrac{x}{\sqrt{1-x^2}}}{-2e^{-2y} + 2\cos y}$

10. Consider a differential equation $\dfrac{dy}{dt} = kt,\ with\, y(0) = 3,\ \dfrac{dy(0)}{dt} = 2$ $then,\ y(t)\ is$

a. $3e^{1.5t}$

b. $3e^{2t/3}$

c. $2e^{1.5t}$

d. $2e^{2t/3}$

11. Consider the following joint distribution of random variables X and Y:

$$f(x,y) = x\left(1+3y^2\right)/4;\ \ 0 < x < 2,\, 0 < y < 1$$

$$f(x,y) = 0;\ \ otherwise$$

The marginal distribution of X is

a. x/4
b. y/4
c. x/2
d. y/6

12. Solve $\lim_{x\to 0}\left(\frac{x}{\sqrt{x+4}-2}\right)$. ________________

13. Find the minima of the function y(x)

$$\frac{dy}{dx} = 2x - e^{-3x} - x^2 - 1$$

a. $-e^{-3}$
b. e^{-3}
c. -3
d. does not exist
e. None

14. Seats for Engineering, Medical and Arts in the university are in the ratio 5 : 7 : 8. There is a proposal to increase these seats by 40%, 50% and 75% respectively. What will be the ratio of increased seats?

a. 2 : 3 : 4
b. 4 : 7 : 12
c. 6 : 8 : 9
d. None

15. Consider the matrix $P = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{-1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$

Which of the following statements are correct ?

a. Determinant of P is 1.
b. P is orthogonal
c. Inverse of P is equal to its transpose.
d. All eigen values of P are real numbers.

16. Two very famous sportsmen Hardik and Krunal happened to be brothers, and played for country K. Hardik teased Shubman, an opponent from country E, “There is no way you are good enough to play for your country.” Shubman replied, “Maybe not, but at least I am the best player in my own family.”

Which one of the following can be inferred from this conversation ?

a. Hardik was known to play better than Shubman.
b. Krunal was known to play better than Hardik.
c. Shubman and Krunal are good friends.
d. Shubman played better than Krunal.

17. Trucks (10m long) and cars (5m long) go on single lane bridge. There must be a gap of at least 25m after each truck and a gap of 20m after each car. Trucks and cars travel at a speed of 48km/hr. If cars and trucks go

alternatively, what is the maximum number of vehicles that can use the bridge in one hour ?

a. 800
b. 960
c. 1400
d. 1600

18. Identify all linear regression models, where $\alpha_i \, and \, \beta$ are model parameters and x_i are variables.

a.
$$y = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix}^T \begin{bmatrix} x_1^2 \\ 1/x_2 \\ x_3 x_2 \end{bmatrix} + \beta + 1$$

b.
$$y = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_1^2 \end{bmatrix}^T \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \beta$$

c. $$y = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_1 \alpha_2 \end{bmatrix}^T \begin{bmatrix} x_1^2 \\ x_2 \\ x_3 x_2 \end{bmatrix} + \beta + 1$$

d. $$y = \begin{bmatrix} 1/\alpha_1 \\ 1/\alpha_2 \\ 1/\alpha_3 \end{bmatrix}^T \begin{bmatrix} x_1^2 \\ x_2 \\ x_3 x_2 \end{bmatrix} + \beta + 1$$

19. The following are the set of data points (x,y) collected in an experiment:

.

$$(1, 2),\ \left(\sqrt{2},\ 4\right),\ \left(\sqrt{3},\ 6\right)$$

The model $y = mx^2$ is proposed to be fitted

to the data points. The value of m is

a. 3
b. 1
c. 4
d. 2

20. Consider the following 3 x 3 matrix **A.**

Mark all the ordered pair (x,y) for which det(A)=0 is

$$A = \begin{bmatrix} x & y & 6 \\ 4 & 4 & 6 \\ 1 & 2 & 3 \end{bmatrix}$$

a. (2, 4)
b. (3, 4)
c. (1, 2)
d. (4, 4)